Exam Mechatronics, 27 February 2014

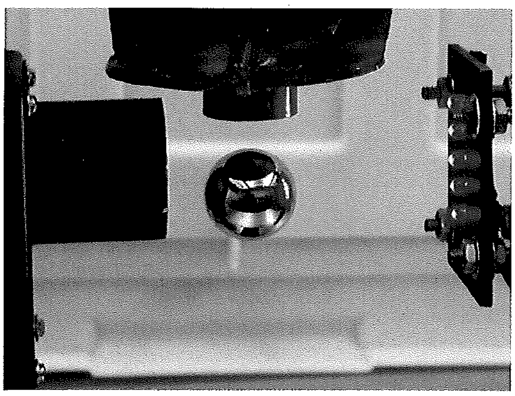
Name: Student ID:

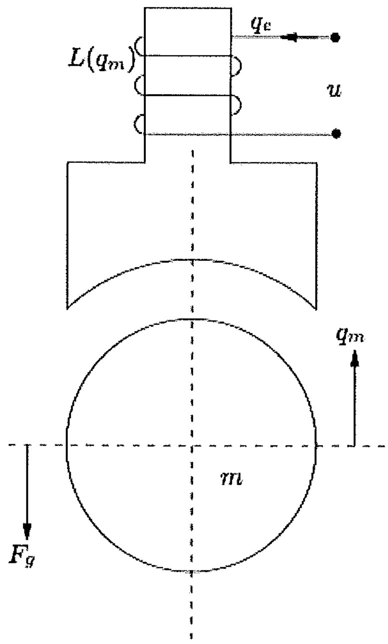
Exercise 1 (25 points)

Note that the Euler-Lagrange equations are given by

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}(q,\dot{q})\right) - \frac{\partial \mathcal{L}}{\partial q}(q,\dot{q}) = -\frac{\partial \mathcal{D}}{\partial \dot{q}} + Gu$$

where $\mathcal{L}$ is the Lagrangian, $\mathcal{D}$ the Rayleigh dissipation function, and G the input matrix.
Consider the levitated ball as a base system that can be found in many mechatronic applications as follows





The system consists of an iron ball with mass m which is placed in a vertical magnetic field which is introduced by an electromagnet. The position of the ball q_m is measured from the surface of the ball to the electromagnet, and $\dot{q}_e$ is the electric current through the inductor $L(q_m)$. Assume that the inductor is linear with respect to the voltage u and that we do not have any parasitic effects. The gravitational force is given as $F_g = mg$. The inductance is defined as $L(q_m) = \frac{\alpha}{(\beta - q_m)^2}$, where α is a design constant, and β the maximal distance of the ball to the electromagnet.

a). Provide the Euler-Lagrange equations of the above system in terms of the ball position ~~x and the beam angle θ~~. Motivate your answer! **(10 points)** Note: if this is a problem, you may consider to determine the equations of motion according to the standard conservation laws. **(in that case 7 points)**.

3 $\mathcal{L}(q,\dot{q}) = \frac{1}{2} m \dot{q}_m^2 + \frac{1}{2} L(q_m) \dot{q}_e^2 - m g q_m$

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$q = (q_m, q_e)$

$\frac{\partial \mathcal{L}}{\partial q_m} = -mg \cancel{\;} \cancel{\;} \dot{q}_e^2 \cdot \frac{\alpha}{(\beta - q_m)^3}$

$\frac{\partial \mathcal{L}}{\partial \dot{q}_m} = m\dot{q}_m$ 3

$\frac{\partial \mathcal{L}}{\partial q_e} = 0$ $\frac{\partial \mathcal{L}}{\partial \dot{q}_e} = \cancel{\;}\frac{\alpha}{(\beta - q_m)^2}\dot{q}_e$

$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_m} = m\ddot{q}_m$, $\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_e} = \frac{\alpha}{(\beta - q_m)^2}\ddot{q}_e + \dot{q}_m\dot{q}_e\frac{\alpha}{(\beta - q_m)^3}$

$$\begin{cases} m\ddot{q}_m + \frac{\alpha}{(\beta - q_m)^3}\dot{q}_e^2 + mg = 0 \\ \frac{\alpha}{(\beta - q_m)^2}\ddot{q}_e + \frac{\alpha}{(\beta - q_m)^3}\dot{q}_m\dot{q}_e = u \end{cases}$$ 4

b). How many states are minimally needed to describe the above system provided that we are interested in the position q_m? Motivate your answer! **(5 points)**.

In fact, there are 2 energy storing elements (inductor, mass), but coupling introduces q_m as well. 3

2

Hence, 3 ~~even~~ states.

c). Provide state space equations of the system. Motivate your answer! **(5 points)**.

$x_1 = q_m, \quad x_2 = \dot{q}_m, \quad x_3 = \dot{q}_e$ 2

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{\alpha}{(\beta - q_m)^3 m} x_3^2 - g$$

3

$$\dot{x}_3 = -\frac{1}{\beta - q_m} x_2 x_3 + u$$

d). Provide state space equations of the system in terms of the position q_m, the momenta p_m and the flux ψ_e. If you did not use them for your answer to c), how are these descriptions related to each other? Are there other state space descriptions possible? Motivate your answer! **(5 points)**.

$$\begin{cases} \dot{q}_m = \frac{1}{m} p_m \\ \dot{p}_m = \frac{\beta - q_m}{\alpha} p_e^2 + mg \\ \dot{\psi}_e = u \end{cases}$$

2

$p_m = m\dot{q}_m$

$\psi_e = L(q_m)\dot{q}_e$

transformation

Hence with $z = (q_m, p_m, \psi_e)$ and $x = (q_m, \dot{q}_m, \dot{q}_e)$ we have

$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ m x_2 \\ \left(\frac{\alpha}{(\beta - x_1)^2}\right) x_3 \end{pmatrix}$$

Others also possible, 2 any non-singular transformation will do.

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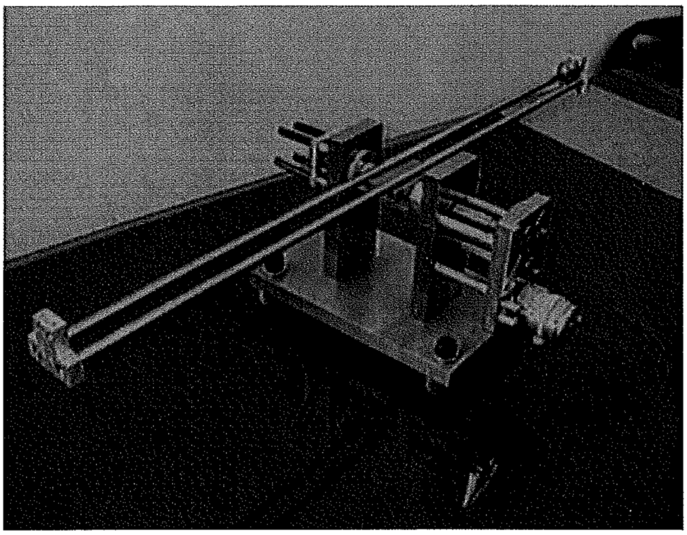
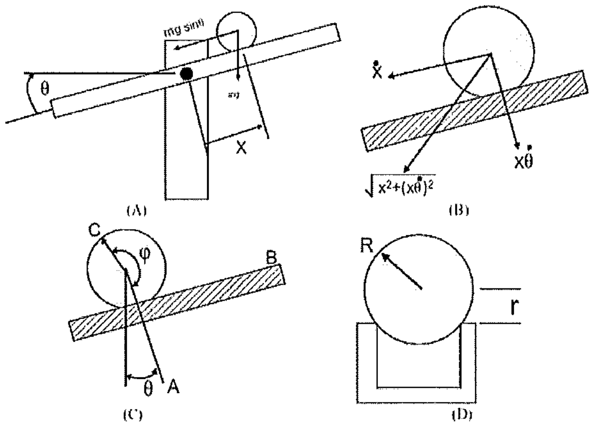
Exercise 2 (15 points)

Consider a ball and beam system incorporated in many mechatronic systems where a balance is necessary. The actuation is done by a motor in the middle of the beam. See the system below





m is the ball mass, x is the translational position (with positive direction pointing downward), θ is the angle of the beam shaft (which can be controlled by the motor). Furthermore, the gravitational constant is given by g, I_b is the moment of inertia of the beam, and r is the effective radius of the ball in the shaft.

The (very) simplified equation of motion is given by

$$\left(m + \frac{I_b}{r}\right)\ddot{x} = mg\theta$$

a). Provide the transfer function from the angle of the beam θ to the position of the ball x. Analyze the stability of the system. **(3 points)**

$$\left(m + \frac{I_b}{r}\right) X(s) s^2 = mg\,\Theta(s)$$

$$\frac{X(s)}{\Theta(s)} = \frac{mg}{\left(m + \frac{I_b}{r}\right)s^2}$$

$s = 0$ marginally stable

b). For controller design we have to consider the discretization. Apply the bilinear and the backward Euler approximations given on page 4, and determine the corresponding difference equations! **(6 points)**.

$$\frac{mg}{\left(m+\frac{I_b}{r}\right)s^2} \Rightarrow$$

$$\frac{mg}{\left(m+\frac{I_b}{r}\right)} \cdot \frac{1}{\frac{1}{T^2}\left(z^{-2}-2z^{-1}+1\right)} = \frac{X(z)}{\theta(z)}$$

$$mg\,\theta(k) = \left(m+\frac{I_b}{r}\right)\frac{1}{T^2}\left(X(k-2)-2X(k-1)+X(k)\right)$$

$$\frac{mg\,z^2}{\left(m+\frac{I_b}{r}\right)\frac{1}{T^2}\left(1-2z+z^2\right)}$$

$$s \rightarrow \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}$$

$$s \rightarrow \frac{1}{T}\left(1-z^{-1}\right)$$

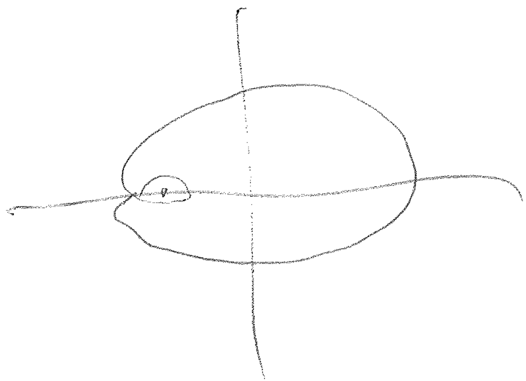
c). Analyze the stability of the discretized systems. Are the stability properties of the original continuous time system preserved? Does the choice for T make a difference? Motivate your answer! **(6 points)**.

char eq. $\left(m + \frac{I_b}{r}\right)\frac{1}{T^2}\left(\cancel{z^{k}}\,1 - 2z^{k+1} + \cancel{z^{k+2}}\right) = 0$

$z^{k+1} = 1$

$T \neq 0$

Marginally stable



ANSWER

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Exercise 3 (30 points)

This exercise concerns stability of systems with delay. The open-loop transfer function is denoted by T_{OL}. The phase margin $\gamma = \pi + \angle T_{OL}(j\omega_g)$, and the gain margin $k_g = \frac{1}{|Re[T_{OL}(j\omega_p)]|}$ are two measures that are derived from the Nyquist criterion, in order to answer if the system is stable and how far the system is from stability. Figure 13.8 shows the polar plot of an open-loop transfer function marked with the bold line. The figure shows the frequency ω_g where the open-loop transfer function magnitude become unity, and the frequency ω_p for which the phase angle of the open-loop transfer function is $-\pi$.

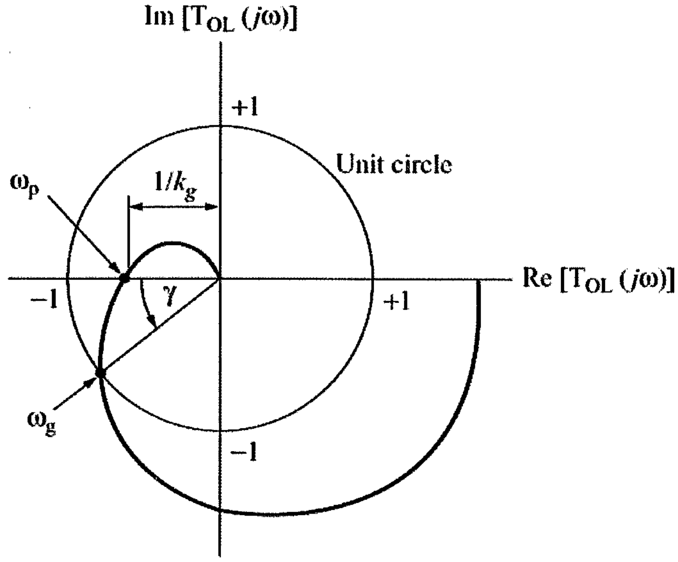


Figure 13.8. Graphical interpretation of stability margins.

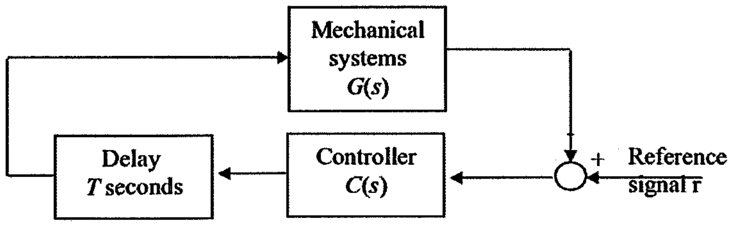
γ·

a). Explain the Nyquist stability criterion. **(5 p)**

See your textbook "Dynamic Modeling ~~of~~ and Control of Engineering Systems" page 339:

If an open-loop transfer function $T_{OL}(s)$ has k poles in the right half of the complex plane, then for stability of the closed-loop system the polar plot of the open-loop ~~system~~ transfer function must encircle the point $(-1, 0j)$ k times in the clockwise direction.

or The number of unstable closed loop poles (Z) is equal to the number of unstable open-loop poles (P) plus the number of clockwise encirclements (N) of the point $(-1, j0)$ of the Nyquist plot of $T_{OL}(s)$.



b).

Suppose the transfer function of the mechanical system is given by $G(s) = \frac{1}{s^2+\sqrt{12}s+3}$, the controller is given by $C(s) = \sqrt{36}$ and the time delay is $T = 4$ seconds. Recall that the open-loop transfer function without delay is given by $T_{OL}(s) = G(s)C(s)$, and that the open-loop transfer function with the time delay is given by $T_{OL}(s) = G(s)C(s)e^{-sT}$.

1) Determine ω_g, i.e. determine at what frequency the open-loop transfer function magnitude is equal to one. 2) Why is the value of ω_g independent of the time delay? **(8 p)**

1) $$1 = |T_{OL}(i\omega_g)| = \frac{\sqrt{36}}{|(i\omega_g)^2 + \sqrt{12}\,i\omega_g + 3|} \quad 2p$$

$$= \frac{\sqrt{36}}{|3-\omega_g^2 + \sqrt{12}\,i\omega_g|}$$

$$= \frac{\sqrt{36}}{\sqrt{(3-\omega_g^2)^2 + 12\omega_g^2}}$$

$$\Rightarrow \omega_g^4 + 6\omega_g^2 - 27 = 0 \quad 2p$$

$$\Rightarrow (\omega_g^2+9)(\omega_g^2-3) = 0$$

must choose the positive solution, thus 2p

$$\underline{\underline{\omega_g = \sqrt{3}}}$$

2p { 2) Independent of timedelay T because $|e^{-i\omega T}| = 1$ for all T.

c). Consider the same system as in b). Based on ω_g obtained in b), determine the phase margin γ for the open-loop system with the time delay. (**8 p**)

2p $$\gamma = \pi + \angle T(i\omega_g)$$

$$= \pi + \angle \frac{\sqrt{36}}{\cancel{(i\sqrt{3})^2} + i\sqrt{3}\sqrt{12} \cancel{+3}} \cdot e^{-i\sqrt{3}4}$$

2p $$= \pi + \angle \frac{\sqrt{36}}{i\sqrt{3}\sqrt{12}} e^{-i\sqrt{3}4}$$

$$= \pi + \angle\sqrt{36} + \angle e^{-i\sqrt{3}4} - \angle i\sqrt{3}\sqrt{12}$$

2p $$= \pi + \tan^{-1}\left(\frac{0}{\sqrt{36}}\right) - \sqrt{3}\cdot 4 - \tan^{-1}\left(\frac{\sqrt{3}\cdot\sqrt{12}}{0}\right)$$

$$= \pi + \tan^{-1}(0) - \sqrt{3}\cdot 4 - \tan^{-1}(\infty)$$

2p $$= \pi + 0 \underbrace{- \sqrt{3}\cdot 4}_{< -\pi} - \frac{\pi}{2} < 0$$

$$\approx -5{,}35$$

d). Based on your answer in c), determine if the closed-loop system with time delay is stable. Motivate your answer.(**5 p**)

knows
2p $|\arg T_{oL}| < \pi$
or $\gamma > 0$

3p right conclusion for the ans in c)

The system is not stable because $\gamma < 0$.

(This means that the point $(-1, j0)$ in Fig 13.8 has been encircled. According to the Nyquist criterion the closed-loop system is then unstable)

e). Determine the critical time delay where the system remains stable, i.e. for what T do we have $\gamma = 0$? (**4 p**)

1p $\angle T_{oL}(i\sqrt{3})e^{-iT\sqrt{3}} = -\pi$

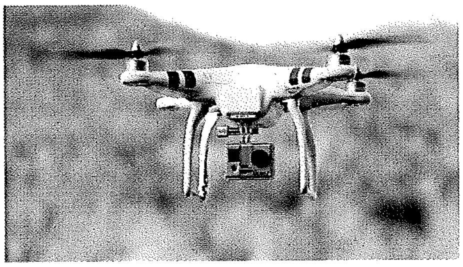
2p $-\frac{\pi}{2} - T\sqrt{3} = -\pi$

$$T = \frac{\pi}{2\sqrt{3}} \simeq 0{,}90$$

1p

Exercise 4 (15 points)

Consider a drone as is used last week by a guy who made a movie from the Python in the Efteling (the Python is a rollercoaster) from the air. Though it was formally not allowed to do this (safety), the Efteling likes the movie. In general, drones can also be used for surveillance in e.g. hazardous environments. Often quadrobots are used for such task, i.e., a system with 4 rotors that allows the system to hover above the object it is inspecting.



a). Identify two possible user demands, two possible functional requirements (FR) and two possible design parameters (DP). **(8 points)**

User demands: - it can ~~hoover~~ fly/hoover without a cable
- it can make videos
- it should be safe
- ....

2

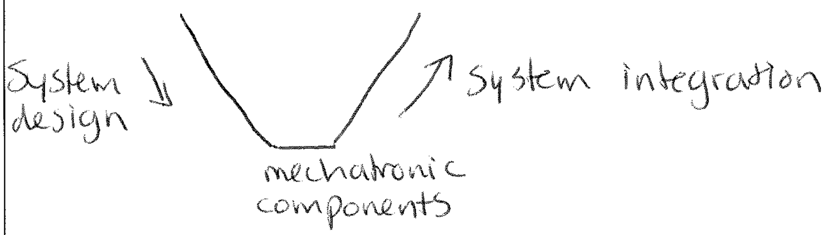
FR: - low use of battery, i.e. less than X watts
- it should be able to hang in one place without too many vibrations, i.e., vibrations amplitude less than X cm., and frequency less than Y Hz.
- ....

3

DP: - use of lightweight rotors that are able to fly it.
- motors can be designed to fullfill FR.
- material and shape of the drone

3

b). Provide the blocks for the Mechatronic design cycle (or V diagram) for the design of a drone for inspection purposes. **(7 points)**



System design

Requirements : User demands, FR

Specifications : DP

System design : similar to mech. diagram of Iserman.

Modeling & simulation : test if design is suitable by simulating elements (motor, rotors, etc.) and overall system.

Component design : each block, rotor, motor, body, camera, ... is designed.

Prototype : build components in laboratory tests.

Mech. comp. — Integrate ~~components coupling~~ considerations digital/analogue

System integration

Comp: Testing : Analysis and testing of prototypes.

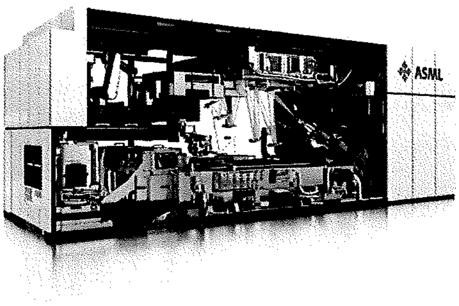
System integration : coupling of components, control algorithm integrated in overall design and analyses.

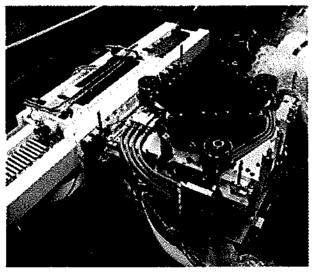
System testing : test if FR is fulfilled ~~and~~ and user demands (does is hang/hover still? test reliability)

Field testing : test drone flying in a field/wood, etc.

Exercise 5 (15 points)

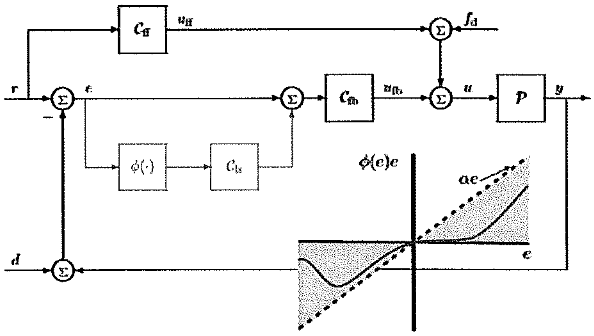
Consider the wafer steppers built by ASML, i.e.,

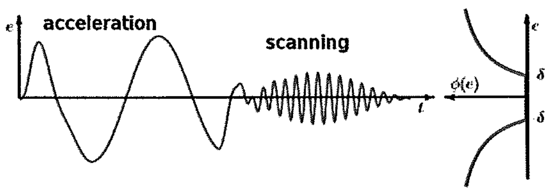




where the right picture shows the long stroke wafer stage, of which the motion needs to be controlled at nanometer level.

a). Recall that the precision positioning system at first is able to accelerate, and when it goes to the scanning phase, it needs to be more precise, and thus have less problems with noise. Now consider the following control scheme with a deadzone nonlinearity $\phi(e)$, and the error response, respectively.

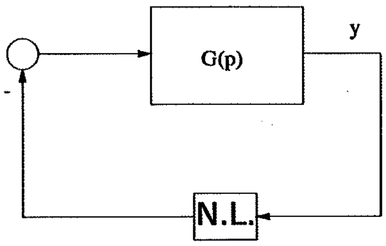


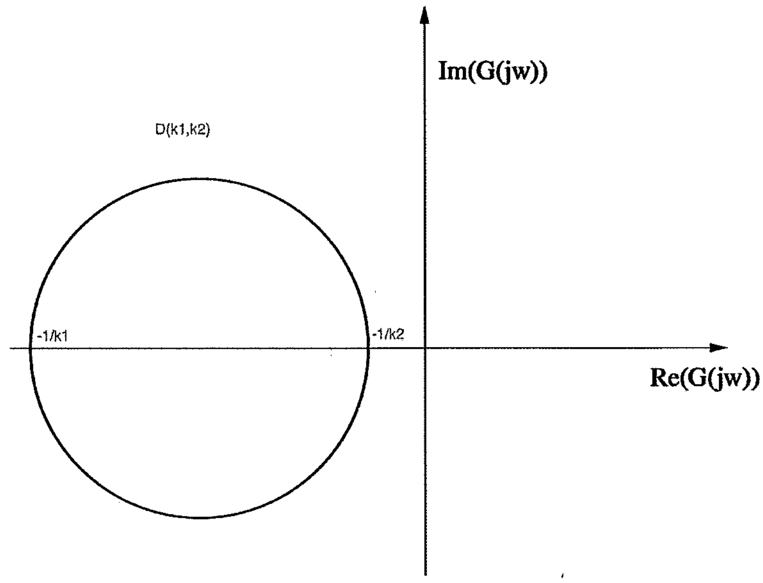


The first part of the error response is the acceleration phase, and the 2nd part the scanning phase. Why is the deadzone nonlinearity useful? **(5 points)**.

The deadzone ensures that when the scanning starts, the ~~machine is~~ system is more precise than before (but slower than before), and that the noise is not amplified when not allowed in the scanning phase

b). Now consider the following scheme (left figure) of a linear plant with a nonlinearity (the bottom block) in the loop. Here N.L. is the nonlinear function ϕ.





The bottom figure represents a circle as can be used in the Circle criterion (which is an extension of the Nyquist criterion). Please provide the Circle criterium with help of this picture. **(10 points)**.

Consider the system $G(p)$ in the loop with φ, then
If A has no eigenvalues on the $j\omega$-axis and ρ
2 eigenvalues in the RHP, and φ belongs to sector $[k_1, k_2]$, then one of the following holds:
2 - if $0 < k_1 \leq k_2$, and the Nyquist plot ~~does not~~ of G does not enter D and encircles it ρ times anti-clockwise
2 - if $0 = k_1 < k_2$, and the Nyquist plot of G stays to the right of $-1/k_2$
2 - if $k_1 < 0 < k_2$, the Nyquist plot of G stays inside D
2 - if $k_1 < k_2 < 0$, and the Nyquist plot of G does not enter $D(-k_1, -k_2)$ and encircles it ρ times anti clockwise
Then 0 is globally asymptotically stable.

END EXAM